RADIOSITY BASED MODEL FOR TERRAIN EFFECTS ON MULTI-ANGULAR VIEWS

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Abstract The Multi-Angular Imaging Spectroradiometer (MISR) will image the Earth from 9 viewing angles and derive atmospheric parameters and surface reflectance.We investigate the effects of reflections within a rough terrain on reflectance retrievals with and without terrain correction. A simple wedge model is used to compute the surface radiosities as a function of solar illumination direction. For the 9 MISR view directions we compute the apparent reflectance and compare the results with a single scattering model. For a range of slopes and flat surface reflectances we compute how much the apparent reflectance changes.

1 Introduction

In mountainous terrain, reflections from adjacent slopes can change the apparent reflectance of a pixel. Very little has been written about this effect because effects of slope angles and terrain height effects on the path radiance are more significant in most cases.

One group of researchers, Dozier and Frew (1989), have computed view factors to adjacent terrain and sky to model radiative processes in snow covered terrain. They give similar expressions to the radiosity formulation except that the light is collected only from adjacent surfaces. They neglect multiple reflections between the slopes. A similar model was developed by Proy, Tanre and Deschamps (1989). For a few selected points in a DEM. They found that a large fraction of the radiance in a shadowed pixel is due to adjacent terrain reflections in the near IR for vegetation (17 to 33 %) and in the VIS/NIR for snow (27 to 48 %). In a paper by Kawata et al. (1992) a method using backward Monte Carlo raytracing is used to compute the radiance due to reflections from adjacent terrain. They found that the effect is negligible for sunlit slopes and significant for shaded slopes and surface albedos of 0.5 to 0.8. The scattered diffuse skylight can be as large as 10 to 25 % of the total radiance in rough terrain and reflections from adjacent terrain can be significant for large sun zenith angles (> 40 degrees). All three groups algorithms are computationally expensive but give some insight into the magnitude of the effect.

To get more insight into the effect of reflected light from adjacent terrain we developed a simple wedge model. It does not represent all the possible effects such as BRDF of the surfaces and scattered skylight effects, but does include multiple reflections and is very fast to compute.

2 The Wedge Model

In Borel and Gerstl (1994) we describe a wedge model in the context of nonlinear mixing models. It is quite simple to expand the model to include arbitrary illumination and viewing angles. Using Hottel's crossed string method it is possible to compute view factors between the illuminated and shaded parts of the wedge and to write the radiosity equations. After the radiosities are computed using an iterative scheme, we compute the visibility of the facets from a given view direction and compute the bidirectional reflectance factor (BRF) which is given by the sum of the products of the radiosities times the visibility divided by the incident energy. The model is quite simple, but allows us to rapidly compute the effects of multiple scattering within a rough surface as a function of illumination and view direction. To keep the notation simple we restrict the illumination angle which is measured from the x axis to angles between 0 and $\frac{\pi}{2}$.

2.1 Wedge Geometry

The geometry of a wedge is shown in Fig. 1. We note that the points on the wedge are given by : $\vec{A} = (-w\cos\theta, w\sin\theta)$, $\vec{B} = (0,0)$, $\vec{C} = (w\cos\theta, w\sin\theta)$, and the shadow point $\vec{S}$ is given by : $\vec{S} = (\vec{A}-\vec{B})f_p$ where :

$$f_p = \begin{cases} \frac{\sin\theta\cos\theta_0 - \cos\theta\sin\theta_0}{\sin\theta} & 0 < \theta_0 < \theta \\ 0 & \theta < \theta_0 < \frac{\pi}{2} \end{cases} .$$

Thus we are only considering illumination angles from $0 < \theta_0 < \frac{\pi}{2}$ so that shadows can only occur on facet 1. The slope angle is θ and the illumination angle θ_0 is measured from the horizon.

The distances between points or "string lengths" are given by :

$$\overline{AB} = |\vec{A} - \vec{B}| = w,$$
$$\overline{BC} = |\vec{B} - \vec{C}| = w,$$
$$\overline{AC} = |\vec{A} - \vec{C}| = 2w\cos\theta,$$
$$\overline{AS} = |\vec{A} - \vec{S}| = (1 - f_p)w,$$
$$\overline{BS} = |\vec{B} - \vec{S}| = f_p w,$$
$$\overline{CS} = |\vec{C} - \vec{S}|.$$

2.2 View Factors

The view factors can be computed using Hottel's crossed string method, Hottel and Sarofim (1967). An example of the method is shown in Fig. 2. The view factor between two infinite strips with areas S_1 and S_2 is given by :

$$S_1F_{12} = \frac{\overline{AD} + \overline{BC} - \overline{AC} - \overline{BD}}{2} ,$$

where $\overline{AD}$ is the length between the points A and D, etc.

We list now the view factors between all facets ($1a$, $1b$, 2 and 3). Note that view factors from and to facet $1b$ exist only if there is a shadow point $\vec{S}$:

$$F_{1a2} = \frac{\overline{AB} + \overline{CS} - \overline{AC} - \overline{BS}}{2w(1 - f_p)},$$

$$F_{1b2} = \frac{\overline{BS} + \overline{BC} - \overline{CS}}{2wf_p},\ if \vec{S}\ exists,$$

$$F_{21a} = \frac{\overline{AB} + \overline{CS} - \overline{AC} - \overline{BS}}{2w},$$

$$F_{21b} = \frac{\overline{BC} + \overline{BS} - \overline{CS}}{2w},\ if \vec{S}\ exists,$$

$$F_{1a3} = \frac{\overline{AS} + \overline{AC} - \overline{CS}}{2w(1 - f_p)},$$

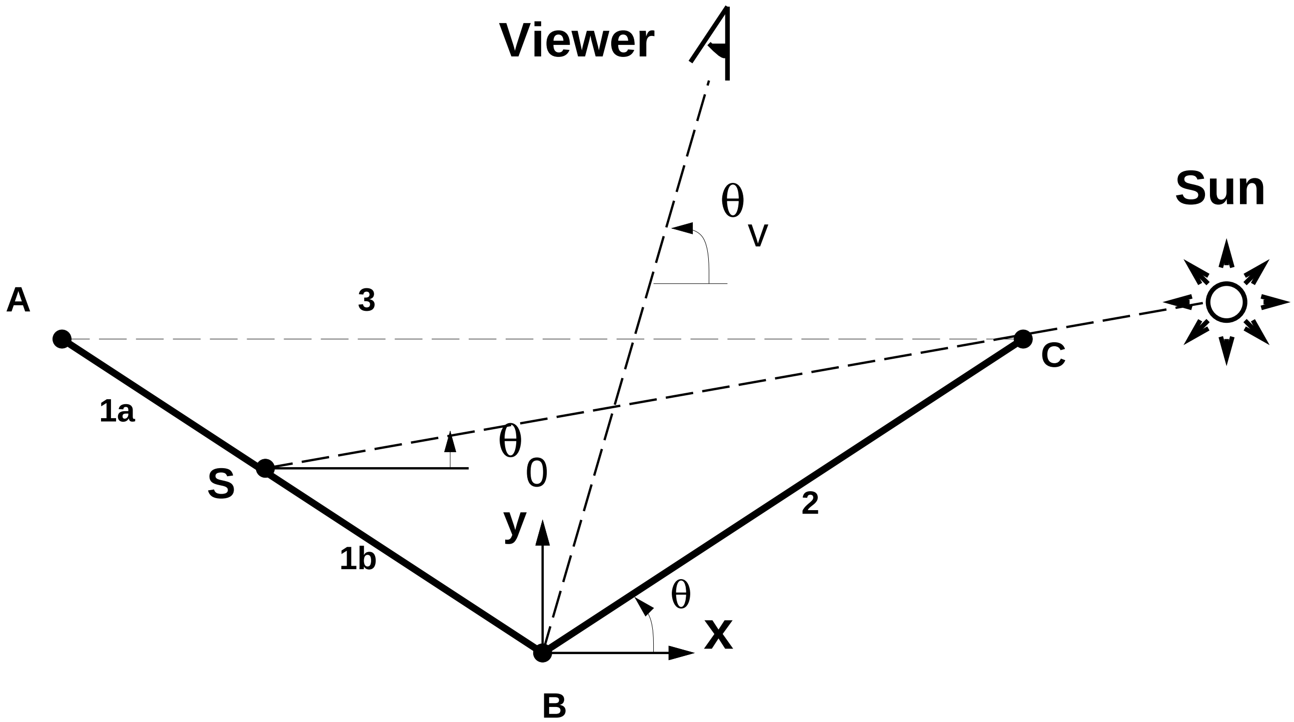

Figure 1: Geometry of a wedge

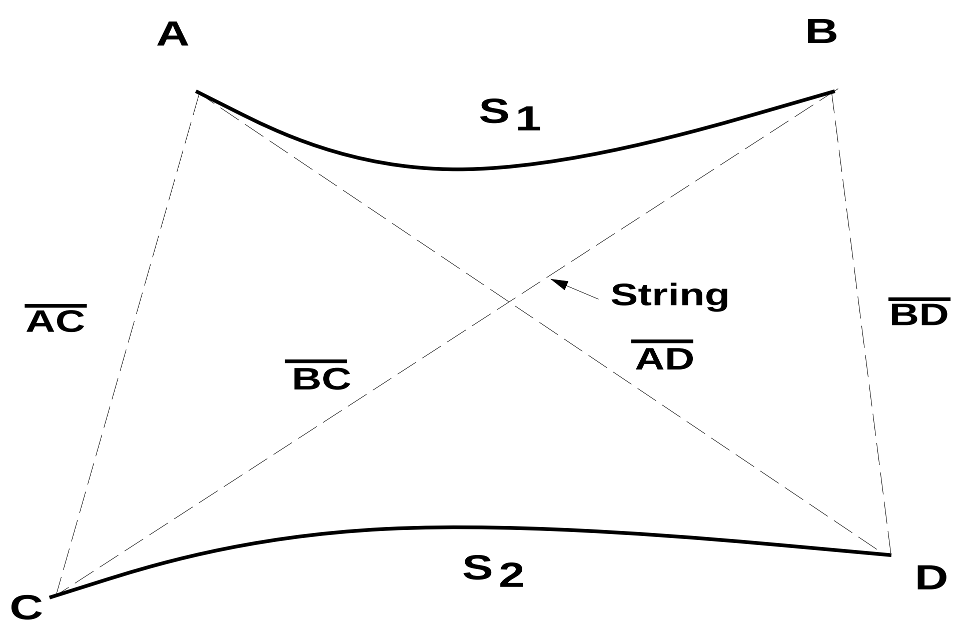

Figure 2: Crossed string method to compute view factors.

$$F_{1b3} = \frac{\overline{AB} + \overline{CS} - \overline{AS} - \overline{BC}}{2wf_p}, \; if \vec{S} \; exists,$$

$$F_{31a} = \frac{\overline{AS} + \overline{AC} - \overline{CS}}{4w \cos\theta},$$

$$F_{31b} = \frac{\overline{AB} + \overline{CS} - \overline{AS} - \overline{BC}}{4w \cos\theta}, \; if \vec{S} \; exists,$$

$$F_{23} = \frac{\overline{BC} + \overline{AC} - \overline{AB}}{2w},$$

and

$$F_{32} = \frac{\overline{BC} + \overline{AC} - \overline{AB}}{4w \cos\theta}.$$

2.3 Radiosity Equations for a Wedge

Facets $1a$ and 2 are illuminated and have initial emission terms :

$$E_{1a} = \rho_1 E_0 \cos(\frac{\pi}{2} - \theta_0 + \theta), \; if \; |\frac{\pi}{2} - \theta_0 + \theta| < \frac{\pi}{2}$$

$$E_2 = \rho_2 E_0 \cos(\frac{\pi}{2} - \theta_0 + \theta), \; if \; |\frac{\pi}{2} - \theta_0 + \theta| < \frac{\pi}{2}$$

where ρ_1 is the reflectance of slope 1 and ρ_2 is the reflectance of slope 2. E_0 is the total incident solar power per unit area in $[Wm^{-2}]$.

The radiosity equations for the three facets can then be written as :

$$B_{1a} = E_{1a} + \rho_1[F_{1a2}B_2 + F_{1a3}B_{sky}],$$

$$B_{1b} = \rho_1[F_{1a2}B_2 + F_{1b3}B_{sky}],$$

and

$$B_2 = E_2 + \rho_2[F_{21a}B_{1a} + F_{21b}B_{1b} + F_{23}B_{sky}].$$

We have included the contributions from sky light (B_{sky}) in these formulas but have set the term to zero for our calculations. The above equations can be solved iteratively and the convergence criterion is met when the root mean square error Δ of successive iterations $(i+1)$ and (i) reaches a lower limit ϵ :

$$\Delta^{(i+1)} = \sqrt{\frac{1}{3}[(B_{1a}^{(i+1)} - B_{1a}^{(i)})^2 + (B_{1b}^{(i+1)} - B_{1b}^{(i)})^2 + (B_2^{(i+1)} - B_2^{(i)})^2]},$$

for $i = 1, 2, 3, ...$

Note that the following weighted sums of view factors add up to unity :

$$F_{1a2}(1 - f_p) + F_{1b2}f_p + F_{1a3}(1 - f_p) + F_{1b3}f_p = 1,$$

$$F_{21a} + F_{21b} + F_{23} = 1,$$

and

$$F_{31a} + F_{31b} + F_{32} = 1.$$

2.4 BRF of a Wedge

To compute the BRF from the radiosities, we need to compute the visibility of each facet from a given view direction θ_v which is defined as the counter clockwise angle measured from the x axis. The BRF is then given in general by (see Borel and Gerstl (1994)) :

$$\rho_{radiosity}(\theta_0;\theta_v) = \frac{P_{1a}B_{1a} + P_{1b}B_{1b} + P_2 B_2}{E_0}.$$

The visibilities P_i for each facet depend on the slope and viewing angle and can be written as :

$$P_{1a} = \begin{cases} 1 & 0 < \theta_v < \theta_c \\ \frac{1-f_p}{1-f_v} & \theta_c < \theta_v < \theta \\ \frac{1-f_p}{2}[1 + \frac{\tan\theta}{\tan\theta_v}] & \theta < \theta_v < \pi - \theta \\ 0 & \pi - \theta < \theta_v < \pi \end{cases},$$

$$P_{1b} = \begin{cases} 0 & 0 < \theta_v < \theta_c \\ \frac{f_p - f_v}{1-f_v} & \theta_c < \theta_v < \theta \\ \frac{f_p}{2}[1 + \frac{\tan\theta}{\tan\theta_v}] & \theta < \theta_v < \pi - \theta \\ 0 & \pi - \theta < \theta_v < \pi \end{cases},$$

and

$$P_2 = \begin{cases} 0 & 0 < \theta_v < \theta \\ \frac{1}{2}[[1 - \frac{\tan\theta}{\tan\theta_v}] & \theta < \theta_v < \pi - \theta \\ 1 & \pi - \theta < \theta_v < \pi \end{cases}.$$

The angle θ_c is given by :

$$\theta_c = \tan^{-1}[\frac{C_y - S_y}{C_x - S_x}].$$

The variable f_v is given by :

$$f_v = \begin{cases} \frac{\sin\theta\cos\theta_v - \cos\theta sin\theta_v}{\sin\theta} & 0 < \theta_v < \theta \\ 1 - \frac{-\sin\theta\cos\theta_v - \cos\theta sin\theta_v}{\sin\theta} & \pi - \theta < \theta_v < \pi \end{cases}.$$

The projection of the ray from the observer onto the wedge is the :

$$\vec{V} = \begin{cases} (\vec{A} - \vec{B})f_v & 0 < \theta_v < \theta \\ (\vec{C} - \vec{B})f_v & \pi - \theta < \theta_v < \pi \end{cases}.$$

The sum of the visibilities is equal to unity for all view angles from 0 to π.

The BRF of a wedge, neglecting multiple reflections between the facets, is given by :

$$\rho_{single}(\theta_0;\theta_v) = \frac{P_{1a}E_{1a} + P_{1b}E_{1b} + P_2 E_2}{E_0}.$$

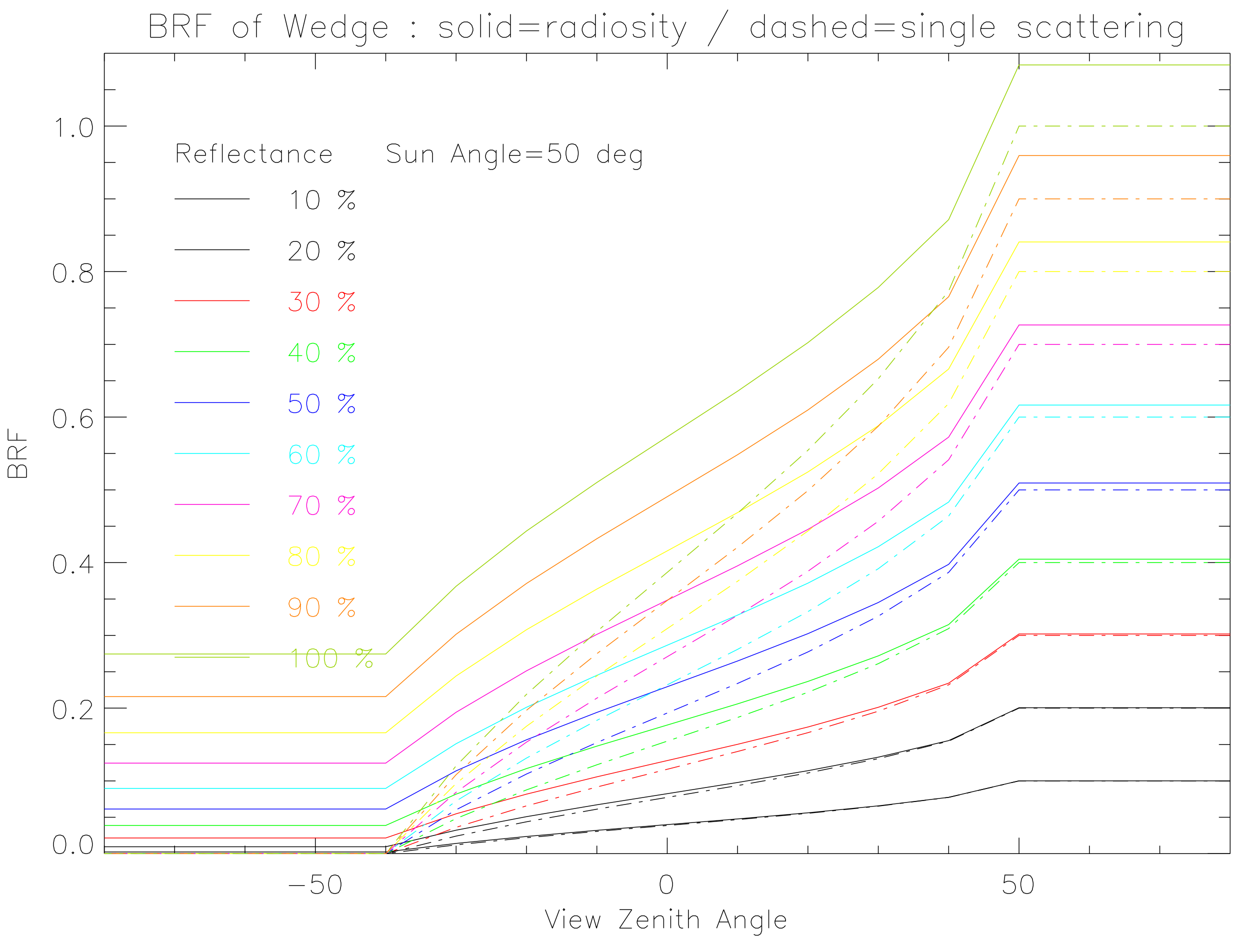

Figure 3: BRF of a wedge with a slope angle $\theta = 50^0$ and a sun angle of $\theta_0 = 40^0$

2.5 Results

The simple wedge model yields some interesting results which seem at first not correct but can be explained. For example if we compute the BRF for a wedge with a slope angle $\theta = 50^0$ and a sun angle of $\theta_0 = 40^0$ we get the BRF shown in Fig. 3. The radiosity computed BRF for viewing zenith angles greater than 50^0 is larger than unity. How can this be? First we note that the light is perpendicularly incident on facet 1, thus the reflectance in the single scattering case must be equal to unity. Second we note that the shaded facet 2 when viewed at angles less than -40^0 is completly dark for the single scattering case but has a BRF of 0.2744 for a 100 % reflecting surface. Third we note that the view factor from facet $1a$ to facet 2 is $F_{1a2} = 0.3064$. Fourth, the amount of light facet $1b$ receives from facet 2 is given by $F_{1a2}B_2 = 0.30640.2744 = 0.0840$ which must be added to the single scattering term. Thus we have shown that the BRF for this case can be indeed be larger than unity.

From Fig. 3 one can see that the effect of scattering of light from the adjacent facet is greatest in the shadowed region ($\theta_v < -40^0$), this is consistent with the results of other studies.

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